DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/ MANAGEMENT/COMMERCIAL PRACTICE — OCTOBER, 2017

ENGINEERING MATHEMATICS – I

[*Time* : 3 hours

(Maximum marks : 100)

PART — A
(Maximum marks : 10)

Marks

I   Answer all questions. Each question carries 2 marks.

1. Prove that $(1 + \cos A)(1 - \cos A) = \sin^2 A$

2. Find the value of $3\sin 15° - 4\sin^3 15°$

3. Find $\dfrac{dy}{dx}$ if $y = x^3 \tan x$.

4. Find the rate of change of volume V with respect to the side of a cube.

5. Find the area of triangle ABC given B = 3cm, C = 2cm and A = 30°

$(5 \times 2 = 10)$

PART — B
(Maximum marks : 30)

II Answer any *five* of the following questions. Each question carries 6 marks.

1. Prove that $\left(\dfrac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1}\right) = \dfrac{1 + \sin\theta}{\cos\theta}$

2. If $\tan A = \dfrac{m}{m + 1}$, $\tan B = \dfrac{1}{2m + 1}$ A and B are acute angles. Prove that A + B = 45°

3. Prove that $\sin 20° . \sin 40° . \sin 80° = \dfrac{\sqrt{3}}{8}$

4. Prove that $R(a^2 + b^2 + c^2) = abc \,(\cot A + \cot B + \cot C)$ where R is radius of circumcircle.

5. Differentiate x^n by method of first principles.

6. A particle moves such that the displacement from a fixed point 'o' is always given by S = 5cos(nt) + 4sin(nt) where n is a constant. Prove that the acceleration varies as its displacement S at the instant.

7. Find the equation to the tangent and normal to the curve $y = 3x^2 + x - 2$ at (1,2).

$(5 \times 6 = 30)$

[2]

Marks

PART — C

(Maximum marks : 60)

(Answer *one* full question from each unit. Each full question carries 15 marks.)

UNIT — I

III (a) Prove that $\left(\dfrac{1+\sin A}{\cos A}\right) = \left(\dfrac{\cos A}{1-\sin A}\right)$ 5

(b) Prove that $\dfrac{\cos(90+A)\,\sec(360+A)\,\tan(180-A)}{\sec(A-720)\,\sin(540+A)\,\cot(A-90)} = 1$ 5

(c) If $\sin A = \dfrac{-4}{5}$ and A lies in third quadrant, find all other trigonometric functions.   5

OR

IV (a) If $\cos A = 3/5$, $\tan B = 5/12$, A and B are acute angles, find the values of $\sin(A+B)$ and $\cos(A-B)$.   6

(b) Prove that $\dfrac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = 2 - \sqrt{3}$ 4

(c) Express $5\sin x - 12\cos x$ in the form $R\sin(x - \propto)$ 5

UNIT — II

V (a) Prove that $\sin 33 + \cos 63 = \cos 3$ 5

(b) Show that $(a-b)\cos\dfrac{C}{2} = c\,\sin\dfrac{A-B}{2}$ 5

(c) Solve triangle ABC, given $a = 2\,cm$ $b = 3\,cm$ $c = 4\,cm$ 5

OR

VI (a) Prove that $\cos\dfrac{\pi}{8} + \cos\dfrac{3\pi}{8} + \cos\dfrac{5\pi}{8} + \cos\dfrac{7\pi}{8} = 0$ 5

(b) Prove that $2\,[bc\cos A + ca\cos B + ab\cos C] = a^2 + b^2 + c^2$ 5

(c) Two angles of a triangular plot of land are $53°17'$ and $67°9'$ and the side between them is measured to be 150m. How many metres of fencing is required to fence the plot ?   5

UNIT — III

VII (a) Evaluate $\underset{x\to 0}{\text{Lt}}\ \dfrac{\sqrt{(1+x)}-1}{x}$ 4

(b) Find $\dfrac{dy}{dx}$, if (i) $y = \dfrac{\cot 11x}{(x^3-1)^2}$ (ii) $(x^2+1)^{10}\sec^5 x$ (3+3)

(c) If $x = a(\theta + \sin\theta)$ $y = a(1 - \cos\theta)$ find $\dfrac{dy}{dx}$ 5

OR

Marks

VIII (a) Find the derivative of cotx using quotient rule. 5

(b) If $y = \sin^{-1} x$ prove that $(1-x^2) \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} = 0$ 5

(c) If x and y are connected by the relation $ax^2 + 2hxy + by^2 = 0$ find $\dfrac{dy}{dx}$. 5

UNIT — IV

IX (a) Show that all the points on the curve $x^3 + y^3 = 3axy$ at which the tangents are parallel to the x-axis lie on the curve, $ay = x^2$. 5

(b) A spherical balloon is inflated by pumping 25cc of gas per second. Find the rate at which the radius of the balloon is increasing when the radius is 15 cm. 5

(c) The deflection of a beam is given by $y = 4x^3 + 9x^2 - 12x + 2$. Find the maximum deflection. 5

OR

X (a) Prove that a rectangle of fixed perimeter has its maximum area when it becomes a square. 5

(b) A circular patch of oil spreads out on water, the area growing at the rate of 6 sq.cm per minute. How fast is the radius increasing when the radius is 2cms.? 5

(c) The distance travelled by a moving body is given by $S = 2t^3 - 9t^2 + 12t + 6$. Find the time when the acceleration is zero. 5